THE LAWVERE CONDITION AND A CLASSIFICATION THEOREM FOR MAL'TSEV-LIKE CATEGORIES

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Mal'tsev, naturally Mal'tsev, and weakly Mal'tsev categories are important classes in categorical algebra with applications in algebraic structures and logic, characterized by distinct properties of their internal relational structures. This talk presents the main results of [3] (see also [2]), which establishes a unified framework classifying these categories through the Lawvere condition – a principle expressing the uniqueness of internal categorical structures such as internal categories and groupoids in terms of underlying reflexive graphs. Each category class corresponds to applying this condition to specific types of spans: all spans, relations (jointly monomorphic spans), or strong relations (jointly strongly monomorphic spans). The framework further incorporates split squares—certain commutative diagrams that generalize pullbacks of split epimorphisms along split epimorphisms—combined via a new compatibility condition analogous to orthogonality between spans and cospans. This leads to a general classification scheme that unifies and extends previous characterizations. The talk will emphasize the conceptual clarity and unifying power of this approach. It builds on earlier related work that developed generalized Mal'tsev properties and strong relations [1,5], and is revisited from a fresh perspective in the recent preprint [4].

References

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[4] Martins-Ferreira, N. On Naturally and Weakly Mal'tsev Categories. arXiv:2508.13315 [math.CT], submitted 18 August 2025.

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