Colimits and monotone-light factorizations

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ABSTRACT

Firstly, precise conditions on how to obtain *very-well-behaved* epireflections are explored and improved from the author's previous papers; meaning that, beginning with a monad and a prefactorization system on a category, is produced a reflection with stable units (stronger than semi-left-exactness, also called admissibility in categorical Galois Theory) and an associated monotone-light factorization. Then, deriving from adjunctions given by left Kan extensions for presheaves, we will show that, for a pseudo-filtered category $\mathbb{J}$ in which every arrow is a monomorphism, the colimit functor on $Set^{\mathbb{J}}$ produces a *very-well-behaved* epireflection; astonishingly, in the very simple case with $\mathbb{J} = \mathbf{2}$, the monotone-light factorization is non-trivial.

References

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