Simplicial Resolutions in Model Categories and a Perspective toward Multisimplicial Constructions in Higher Categories

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Abstract

Model categories provide a natural framework for formalizing homotopy theory within category theory, combining cofibrations, fibrations, and weak equivalences in a unified setting. Within this framework, simplicial resolutions emerge as categorical analogues of projective resolutions, constructed using simplicial kernels and resolving subcategories. This approach generalizes classical homological constructions to non-additive contexts and allows a structured method for building simplicial objects from categorical data.

The construction of simplicial resolutions through simplicial kernels relies on a stepwise process rooted in universal properties and categorical limits. Resolving subcategories play a key role in replacing projective objects and ensuring the existence of appropriate morphisms at each stage of the resolution.

This framework opens a path toward extending the theory to higher categories, where notions of homotopy coherence become central. The formulation of multisimplicial resolutions in higher categorical contexts represents a natural objective, aiming to capture refined homological and homotopical information in a multidimensional setting.

Keywords: model categories; simplicial resolutions; simplicial kernels; resolving subcategories; homotopy theory; higher categories; multisimplicial resolutions; homotopy coherence.

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